

Seasonal Adjustment and Transformations

Destagionalizzazione di serie trasformate

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Riassunto: Il lavoro affronta il problema della destagionalizzazione in presenza di una trasformazione della serie originaria, del tipo di Box-Cox per serie misurate su scala di rapporto, o di Aranda-Ordaz per rapporti di composizione, che mira a garantire il conseguimento dei due requisiti fondamentali: additività e ortogonalità delle componenti. La media a posteriori della serie destagionalizzata non ha in generale una forma esplicita, ma può essere valutata numericamente o mediante simulazione Monte Carlo, tramite l'algoritmo noto come *simulation smoother*. Il lavoro presenta un'applicazione concernente la destagionalizzazione di un indice della produzione industriale in presenza di una trasformazione di Box-Cox di parametro $1/2$ (radice quadrata).

Keywords: Seasonal adjustment, state space models, Kalman filter and smoother.

1. Introduction

Seasonal adjustment is founded upon two basic assumptions: *additivity* and *orthogonality* of the seasonal and non seasonal components. According to Bell and Hillmer (1984, sec. 4.2), “someone who does not want to make these assumptions is working on a different problem”.

This paper considers the situation when the two previous requirements are fulfilled on a scale other than the original scale of measurement and provides a novel model-based solution to the adjustment problem. Seasonal adjustment under transformations has also been addressed by Thomson and Ozaki (2002), who propose ad hoc solutions that aim at enforcing the seasonal balance constraint on the original scale (the seasonal component averages out to zero over a yearly span). Our solution is based instead on the evaluation of the posterior mean of the non seasonal component by numerical integration or by Monte Carlo simulation techniques, based on the simulation smoother (de Jong and Shephard, 1995); thus, the stochastic seasonal balance constraint is assumed to hold on the transformed scale.

2. The basic structural model under transformations

The linear and Gaussian model that we employ for the adjustment is the basic structural model (BSM henceforth, see Harvey, 1989), which postulates an additive and orthogonal decomposition of a time series into unobserved components representing the trend, seasonality and the irregular component.

We assume that the BSM holds for a transformation $u_t(\lambda)$ of the original time series y_t , depending on a single transformation parameter λ . An important case is the Box-Cox

(BC) transformation:

$$u_t(\lambda) = \begin{cases} \frac{y_t^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \ln y_t, & \lambda = 0 \end{cases}$$

see Box and Cox (1964). The above transformation is suitable for series measured on a ratio scale.

For proportions we consider the Aranda-Ordaz (1981, AO henceforth) transformation:

$$u_t(\lambda) = \frac{2 y_t^\lambda - (1 - y_t)^\lambda}{\lambda y_t^\lambda + (1 - y_t)^\lambda} = \frac{2 p_t^\lambda - 1}{\lambda p_t^\lambda + 1},$$

where $p_t = y_t/(1 - y_t)$ denotes the odds. This has several amenable properties: it is invertible and yields the logit transformation, $u_t = \ln(y_t/(1 - y_t))$, for $\lambda \rightarrow 0$ and the untransformed series for $\lambda = 1$, $u(1) = 2(2y_t - 1)$.

The BSM for the transformed series is formulated as follows:

$$u_t(\lambda) = \mu_t + \gamma_t + \delta' x_t + \epsilon_t, \quad t = 1, \dots, T,$$

where μ_t is the trend component, γ_t is the seasonal component, the x_t 's are appropriate regressors that account for calendar effects, namely working days, moving festivals (Easter) and the length of the month, and $\epsilon_t \sim \text{NID}(0, \sigma_\epsilon^2)$ is the irregular component.

The trend component has a local linear representation:

$$\begin{aligned} \mu_{t+1} &= \mu_t + \beta_t + \eta_t, & \eta_t &\sim \text{NID}(0, \sigma_\eta^2), \\ \beta_{t+1} &= \beta_t + \zeta_t, & \zeta_t &\sim \text{NID}(0, \sigma_\zeta^2), \end{aligned}$$

where β_t is the stochastic slope, that in turn evolves as a random walk; the disturbances η_t , ζ_t , are independent of each other and of any remaining disturbance in the model.

The seasonal component has a trigonometric representation, such that the seasonal effect at time t arises from the combination of six stochastic cycles: $\gamma_t = \sum_{j=1}^6 \gamma_{jt}$, where, for $j = 1, \dots, 5$,

$$\begin{aligned} \gamma_{j,t+1} &= \cos \lambda_j \gamma_{j,t} + \sin \lambda_j \gamma_{j,t}^* + \omega_{j,t} & \omega_{j,t} &\sim \text{NID}(0, \sigma_\omega^2) \\ \gamma_{j,t+1}^* &= -\sin \lambda_j \gamma_{j,t} + \cos \lambda_j \gamma_{j,t}^* + \omega_{j,t}^* & \omega_{j,t}^* &\sim \text{NID}(0, \sigma_\omega^2) \end{aligned}$$

and $\gamma_{6,t+1} = -\gamma_{6t} + \omega_{6t}$, $\omega_{6t}^* \sim \text{NID}(0, \sigma_\omega^2/2)$; $\lambda_j = \frac{2\pi}{12}j$ is the seasonal frequency; The disturbances ω_{jt} and ω_{jt}^* are assumed to be normally and independently distributed with common variance σ_ω^2 . All the disturbances are assumed to be mutually uncorrelated.

3. Seasonal Adjustment under transformation

Writing $u_t(\lambda) = u_t^* + \gamma_t + \beta' x_t$, where $u_t^* = \mu_t + \epsilon_t$ is the seasonally adjusted series on the transformed scale, let us denote by $\tilde{u}_t^* = E(u_t^* | \mathcal{F}_T)$ and $V_t^* = \text{Var}(u_t^* | \mathcal{F}_T)$, respectively the posterior mean and variance of u_t^* , $\mathcal{F}_t$ being the information set at time t . These inferences are delivered by the Kalman filter and smoother applied to the relevant linear state space model.

We define the seasonally adjusted series on the original scale as $y_t^* = u^{-1}(u_t^*)$, where $u^{-1}(\cdot)$ is the inverse transformation, e.g. in the BC case:

$$y_t^* = \begin{cases} (1 + \lambda u_t^*)^{1/\lambda}, & \lambda \neq 0, \\ \exp(u_t^*), & \lambda = 0, \end{cases}$$

whereas for the AO transformation $y_t^* = [1 + \exp(-v_t)]^{-1}$, where $v_t = u_t$ if $\lambda = 0$ and $v_t = \lambda^{-1} \ln [(2 + u_t\lambda)/(2 - u_t\lambda)]$, otherwise.

The estimate of the seasonally adjusted series is thus

$$\tilde{y}_t^* = E(y_t^* | \mathcal{F}_T) = \int u^{-1}(u_t^*) f(u_t^* | \mathcal{F}_T) du_t^*. \quad (1)$$

The above integral does not have a closed form solution, unless $\lambda = 1$ and $\lambda = 0$, for the BC transformation, and $\lambda = 1$ in the AO case. The conditional variance of the estimation error is defined as follows:

$$\text{Var}(y_t^* | \mathcal{F}_T) = \int [u^{-1}(u_t^*) - \tilde{y}_t^*]^2 f(u_t^* | \mathcal{F}_T) du_t^* = E(y_t^{*2} | \mathcal{F}_T) - \tilde{y}_t^{*2}. \quad (2)$$

Again, analytic solutions are available only in the above cases. For general λ there are two possible ways of evaluating these quantities:

- Monte Carlo evaluation using the simulation smoother: the latter is used to draw repeated samples from the conditional distribution of $u^* = \{u_1^*, \dots, u_T^*\}$, given the available observations.
- Numerical integration with respect to the normal density, $f(u_t^* | \mathcal{F}_T)$, whose moments $\tilde{u}_t^*$ and V_t^* are provided by the Kalman filter and smoother.

4. Illustrations

Due to space constraints we provide only one illustrative example dealing with the evaluation of (1) and (2) for the seasonal adjustment of the industrial production index for the Leather and Shoes sector, available for the period 1981.1-2005.2 (source Istat, base 2000 = 100), under the Box-Cox transformation. The official seasonal adjustment uses Tramo-Seats on the untransformed series.

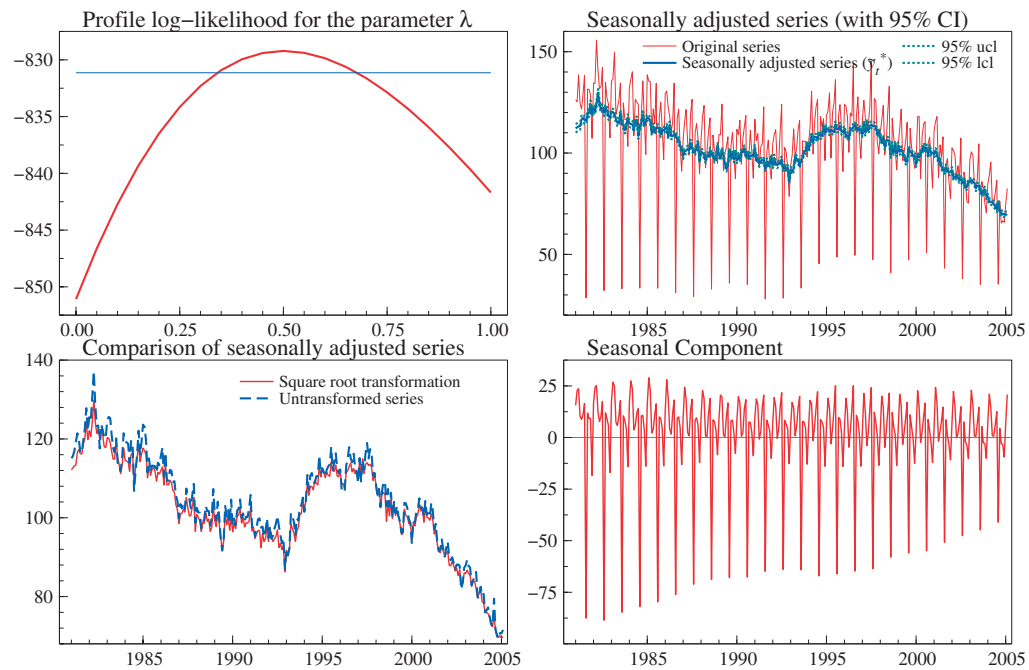
However, the likelihood ratio test of $H_0 : \lambda = 1$ is significant and the maximum likelihood estimate of the transformation parameter is $\tilde{\lambda} = 0.5$, corresponding to the square root transformation. This evidence is in line with that presented in Proietti (1995). The profile likelihood is plotted against λ in the first panel of figure 1.

The Monte Carlo estimates of the seasonally adjusted series, (1), and its variance (2), are computed using the simulation smoother (de Jong and Shephard, 1995) with $M = 5000$ replications. For instance, in order to evaluate $\tilde{y}_t^*$, we draw M independent samples $u_t^{(i)*}, i = 1, \dots, M$, from the conditional distribution $u_t^* | \mathcal{F}_T \sim N(\tilde{u}_t^*, V_t^*)$, which is done recursively by the simulation smoother and compute the average $\frac{1}{M} \sum_{i=1}^M [1 + 0.5 u_t^{(i)*}]^2$.

The point estimates of the seasonally adjusted series are plotted in the right top panel figure 1, along with an approximate 95% confidence interval. These are compared to those of the untransformed series ($\lambda = 1$) in the bottom left panel, whereas the bottom

right panel displays the estimated seasonal component using the MLE $\tilde{\lambda} = 0.5$. The comparison points out that the estimates may differ significantly, i.e. the estimates based on the original series can lie outside the 95% confidence interval for the ones obtained under transformation, and that the former are biased upwards.

Figure 1: *Index of Industrial Production, Sector DC: Leather and Shoes.*



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